Behavior Modeling in Transportation Networks

Lecture Series #2-5 (16:20-16:50)

RL model and Advanced Behavior Modeling

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September 14, 2020

Recall the logit model

Choice probability:

$$P(i) = \frac{e^{\mu v_i}}{\sum_{j \in \mathcal{C}} e^{\mu v_j}} \geq 0$$

$$\sum_{j \in \mathcal{C}} P(i) = 1$$

v_i : deterministic utility of i

$\varepsilon_i \overset{\text{i.i.d.}}{\sim} \text{Gumbel}(0, \mu), \forall i$: error term

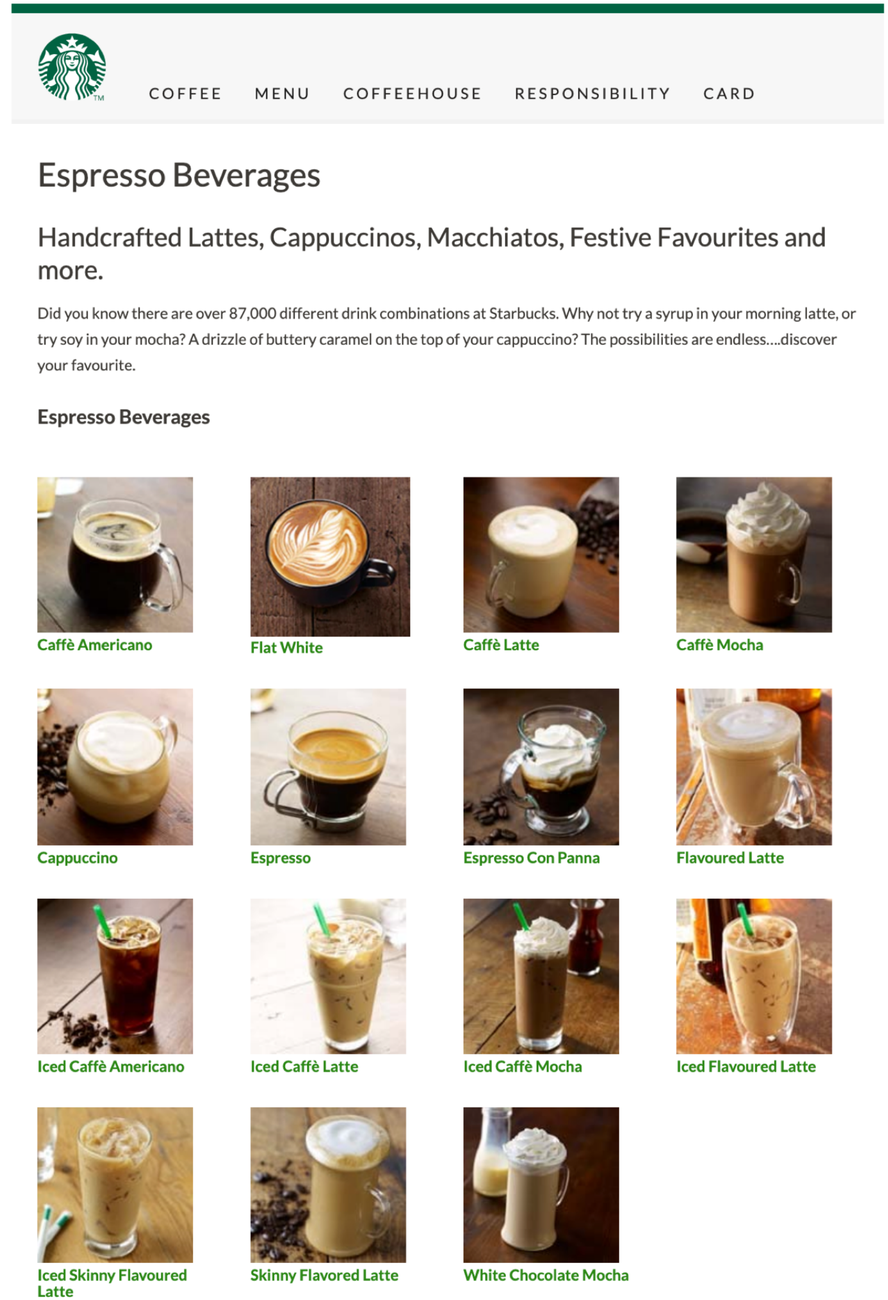
$\mathcal{C}$: choice set

Value of choice situation (**welfare**):

$$V = \mathbb{E}\left[\max_{j \in \mathcal{C}} \{v_j + \varepsilon_j\}\right] = \frac{1}{\mu} \ln \sum_{j \in \mathcal{C}} e^{\mu v_j}$$

"Did you know …

there are over **87,000 different drink combinations** at Starbucks?"



Choice is often **combination of elemental choices**

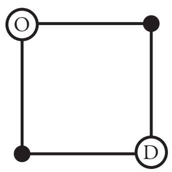
e.g., activity pattern (H, W, O), mode choice (MaaS), tour planning...

Choice set can become huge and is **difficult to define**...

Route choice is a typical example

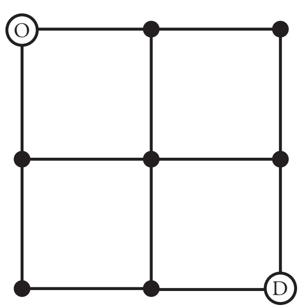
Route = Combination of links

$k = 1$



$|\mathcal{C}| = 2$

$k = 2$



$|\mathcal{C}| = 12$

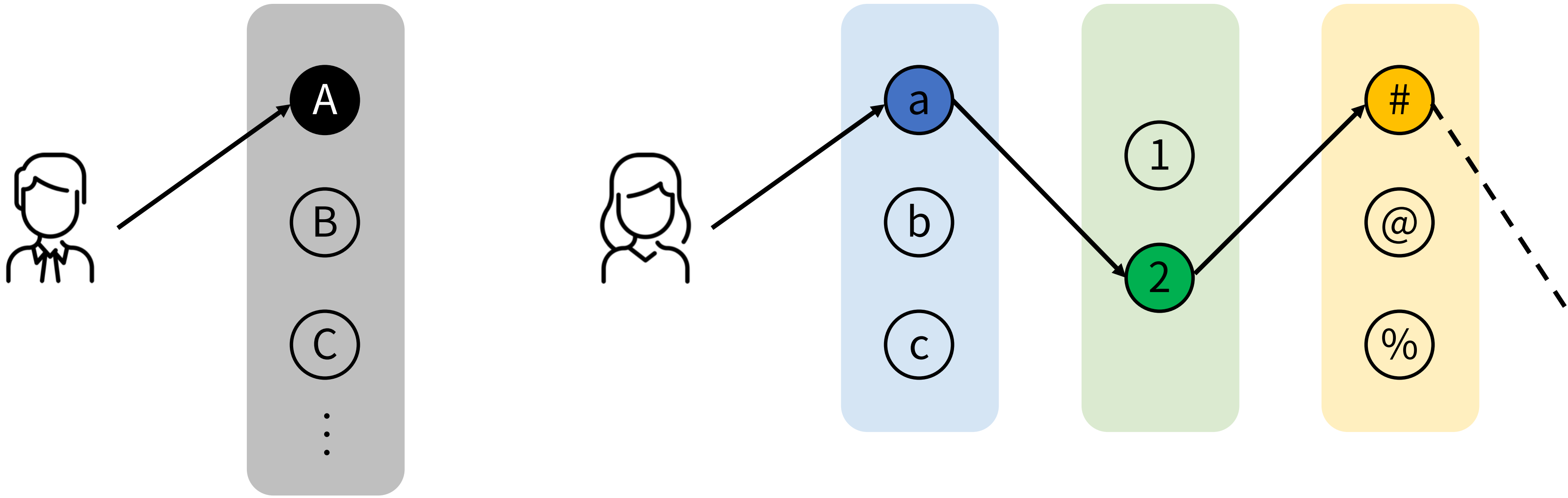
k	simple paths
1	2
2	12
3	184
4	8,512
5	1,262,816
6	575,780,564
7	789,360,053,252
8	3,266,598,486,981,640
9	41,044,208,702,632,496,804
10	1,568,758,030,464,750,013,214,100
11	182,413,291,514,248,049,241,470,885,236
12	64,528,039,343,270,018,963,357,185,158,482,118
13	69,450,664,761,521,361,664,274,701,548,907,358,996,488
14	227,449,714,676,812,739,631,826,459,327,989,863,387,613,323,440
15	2,266,745,568,862,672,746,374,567,396,713,098,934,866,324,885,408,319,028

way more than Starbucks...

What can we do ?

Not combination but **SEQUENCE** of choices !

$$A = [a, 2, \#, \ldots]$$



Choice

Sequence of choices

How to model sequences ?

In the case of route choice, a route r can be described as:

$$r = [a_1, a_2, \ldots, a_J]$$

a sequence of links

Route choice probability:

$$P(r) = \prod_{j=1}^{J-1} p(a_{j+1}|a_j)$$

$p(a_{j+1}|a_j)$: Link choice probability **conditional on the previous link**

$\Rightarrow$ Seems easy⋯, but **what is link choice probability** exactly?

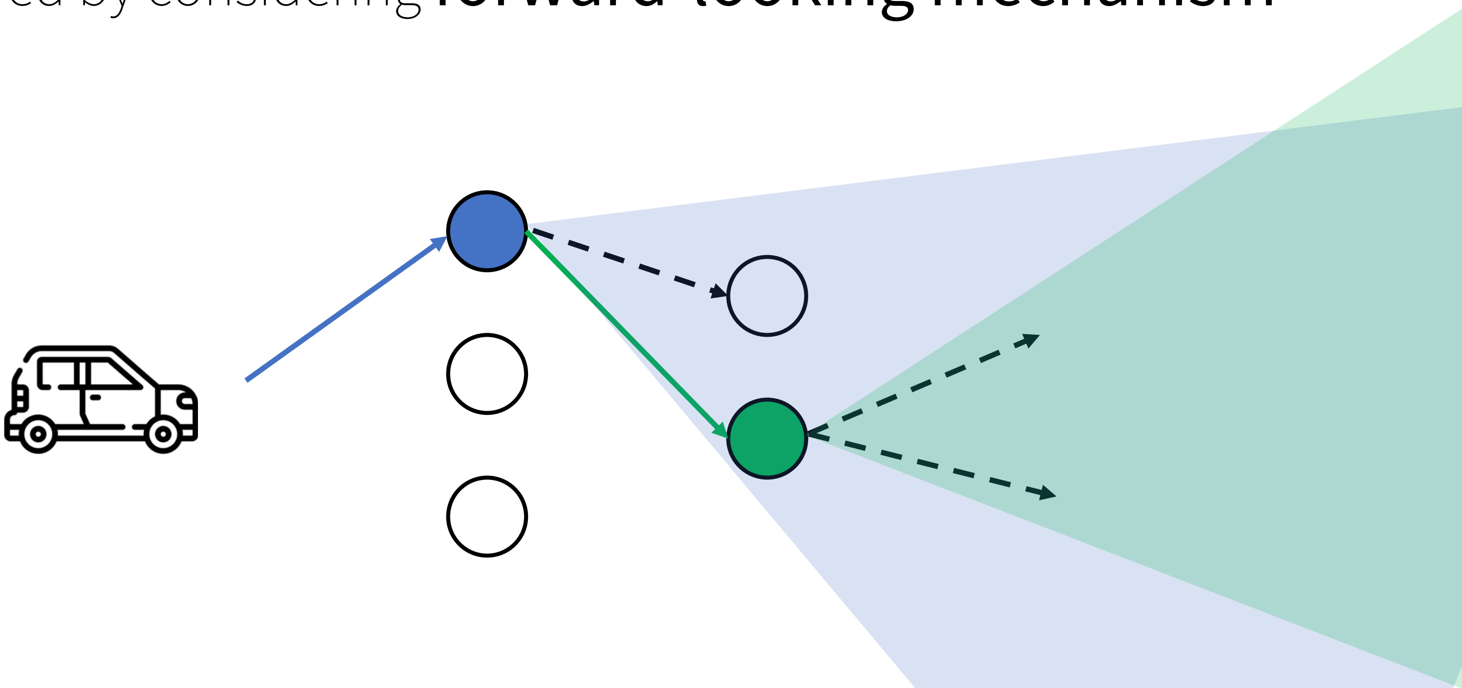
What should be considered is ...

the outcome given by the product of link choice probabilities should be **consistent with the original model**, i.e.,

$$P(r) = \prod_{j=1}^{J-1} p(a_{j+1}|a_j) = P_{\text{Logit}}(r)$$

*when assuming logit model

This is achieved by considering **forward-looking mechanism**

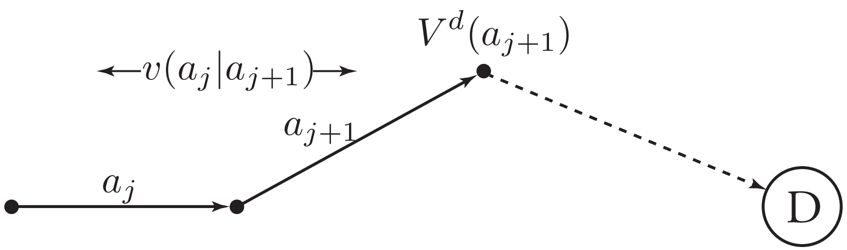


Value function

Goal is modeling

1. Myopic
2. Forward-looking

mechanisms of behavior



v : Link choice utility

V^d : Value function

$$V^d(a_j) = \mathbb{E}\left[\max_{a_{j+1}\in\mathcal{A}(a_j)}\{v(a_{j+1}|a_j) + \varepsilon(a_{j+1}|a_j) + V^d(a_{j+1})\}\right]$$

Random utility

Generalization

c.f. **Shortest Path (SP) problem**:

$$V^d(a_j) = \max_{a_{j+1}\in\mathcal{A}(a_j)}\{v(a_{j+1}|a_j) + V^d(a_{j+1})\}$$

Value function is the **SP cost** from a_j to destination

Gumbel distribution has a nice property:

$$\varepsilon_k \overset{\text{i.i.d.}}{\sim} \text{Gumbel}(0, \mu), \forall k \quad \Rightarrow \quad \max_k \{\eta_k + \varepsilon_k\} \sim \text{Gumbel}(\frac{1}{\mu} \ln \sum_k \mu \eta_k, \mu)$$

Value function is the solution to:

$$V^d(a_j) = \mathbb{E}\left[\max_{a_{j+1} \in \mathcal{A}(a_j)} \{v(a_{j+1}|a_j) + \varepsilon(a_{j+1}|a_j) + V^d(a_{j+1})\} \right]$$

$$= \frac{1}{\mu} \ln \sum_{a_{j+1} \in \mathcal{A}(a_j)} e^{\mu\{v(a_{j+1}|a_j) + V^d(a_{j+1})\}}$$

$$\Leftrightarrow \quad e^{\mu V^d(a_j)} = \sum_{a_{j+1} \in \mathcal{A}(a_j)} e^{\mu v(a_{j+1}|a_j)} e^{\mu V^d(a_{j+1})}$$

a system of linear equations.

(Recurrence relation)

$$\Rightarrow \quad \mathbf{z}^d = \mathbf{W}\mathbf{z}^d + \mathbf{e}^d$$

$$\mathbf{z}^d \equiv [e^{\mu V_k^d}]_{k \in \mathcal{L}} \qquad \mathbf{W} \equiv [e^{\mu v(l|k)}]_{k,l \in \mathcal{L}} \qquad \mathbf{e}^d \equiv [\delta_k^d]_{k \in \mathcal{L}}$$

Value function — Weight incidence matrix — Unit vector

Let's check the Consistency!

Link choice probability is given by:

$$p^d(a_{j+1}|a_j) = \frac{e^{\mu\{v(a_{j+1}|a_j)+V^d(a_{j+1})\}}}{\sum_{a_{j+1}\in\mathcal{A}(a_j)} e^{\mu\{v(a_{j+1}|a_j)+V^d(a_{j+1})\}}} = \frac{W(a_{j+1}|a_j)z^d(a_{j+1})}{z^d(a_j)}$$

*like logit by assuming $U(a_{j+1}|a_j) = \underbrace{v(a_{j+1}|a_j) + V^d(a_{j+1})}_{\text{New deterministic utility}} + \varepsilon(a_{j+1}|a_j)$

Then we have:

$$P^{od}(r) = \frac{W(a_1|o)z^d(a_1)}{z^d(o)} \cdot \frac{W(a_2|a_1)z^d(a_2)}{z^d(a_1)} \cdot \cdots \cdot \frac{W(d|a_J)z^d(d)}{z^d(a_J)}$$

($z^d(d) = 1$)

$$= \frac{\prod_{j=0}^{J} W(a_{j+1}|a_j)}{z^d(o)} = \frac{e^{\mu\sum_{j=0}^{J} v(a_{j+1}|a_j)}}{e^{\mu V^d(o)}} = \frac{e^{\mu v_r}}{\sum_{r'\in\mathcal{R}^{od}} e^{\mu v_{r'}}}$$

Route utility is sum of link utilities

Exp. Max. of **all possible paths**

$$= P_{\text{Logit}}(r|\mathcal{R}^{od})$$

$\Rightarrow$ **Consistent with logit** route choice model with the universal choice set

So, what's the point ?

Now you can model route choice behavior
without explicitly defining choice set

k	simple paths
1	2
2	12
3	184
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15	2,266,745,568,862,672,746,374,567,396,713,098,934,866,324,885,408,319,028

No longer needed!

1. Decompose route choice into **sequential link choices:**

$$P(r) = \prod_{j=1}^{J-1} p(a_{j+1}|a_j)$$

2. Describe forward-looking behavioral mechanism by **value function:**

$$V^d(a_j) = \frac{1}{\mu} \ln \sum_{a_{j+1} \in \mathcal{A}(a_j)} e^{\mu\{v(a_{j+1}|a_j) + V^d(a_{j+1})\}}$$

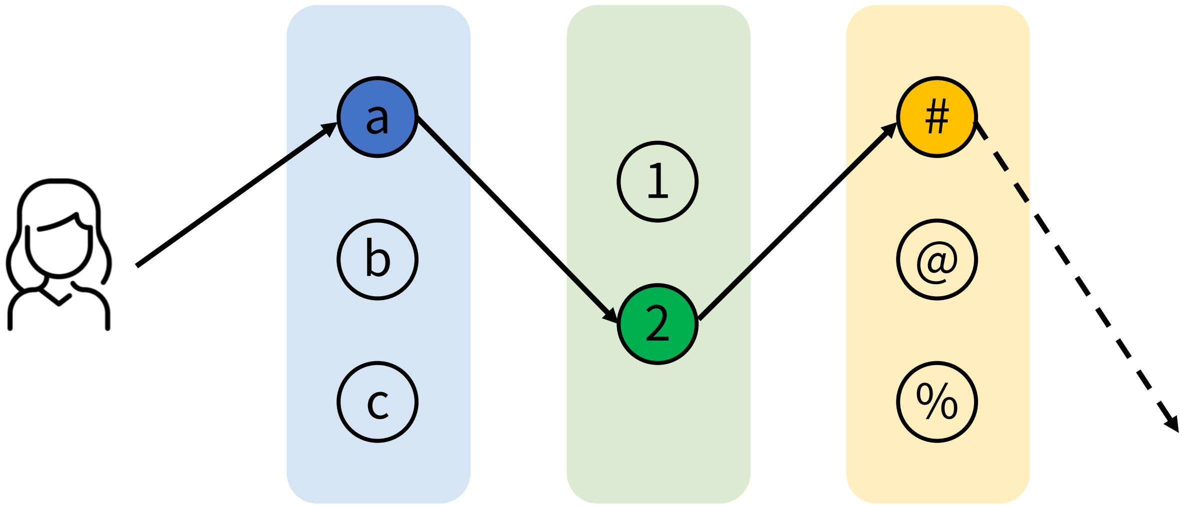
Recursively computed

This (efficient) computational method of modeling is called:

"Recursive Logit (RL) model"

Modeling sequence is something more

than just dealing with the choice set problem.



You can also try other sequences than route choice:

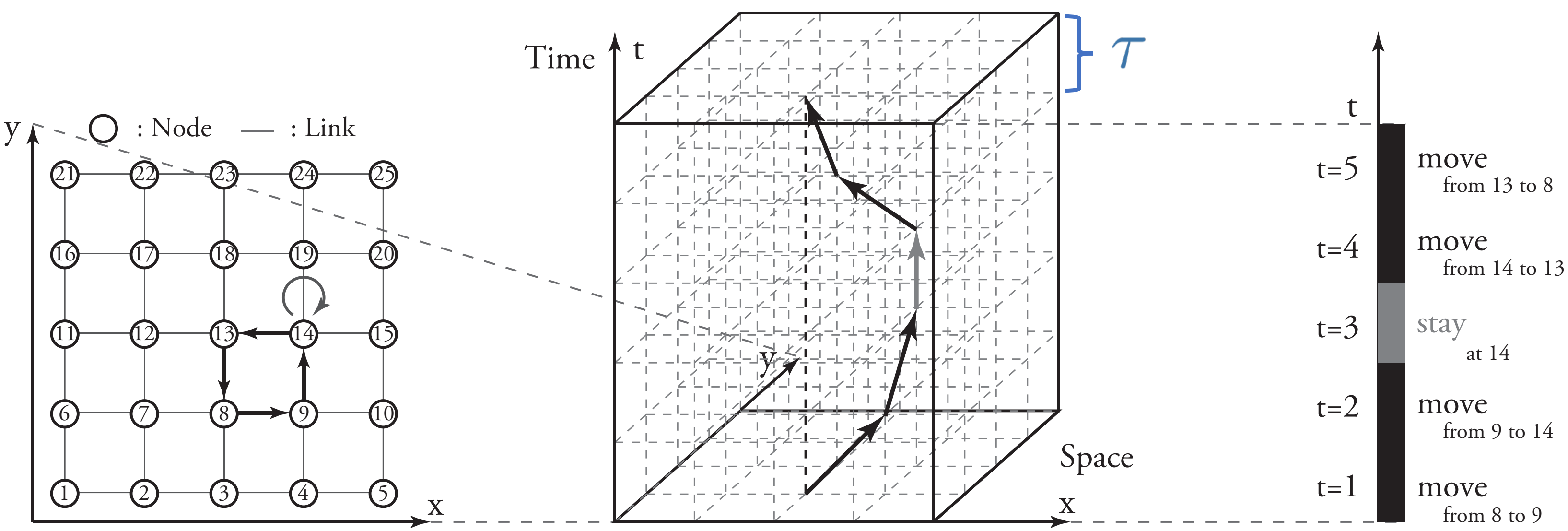
- **Mode choice**: unlinked trip choices
- **Tour choice**: sequential destination choices
- **Timing choice**: “action (do)/stay (don’t)” choice at every period
- etc.

Activity path modeling

Route choice can be interpreted as **sequential space choice**

$$r = [a_1, a_2, \dots, a_J]$$

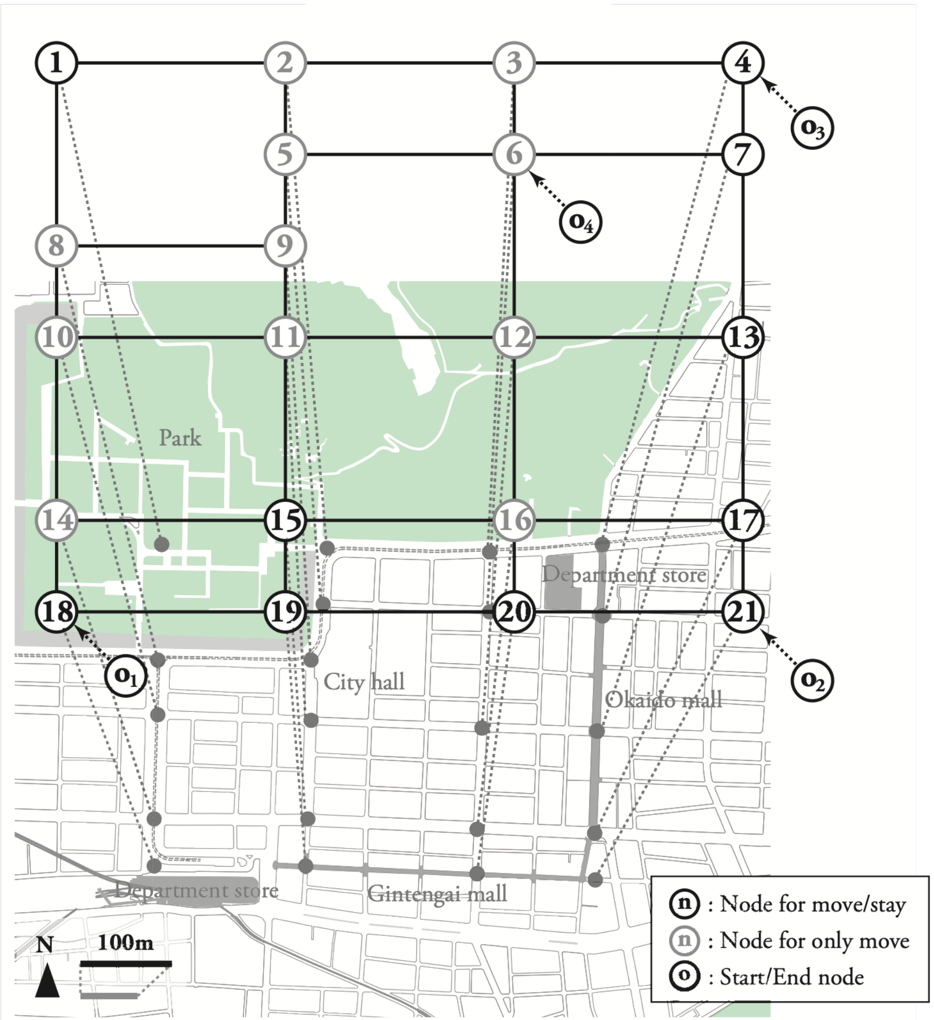
Introducing a **fixed time-interval (τ)** for decision making yields:

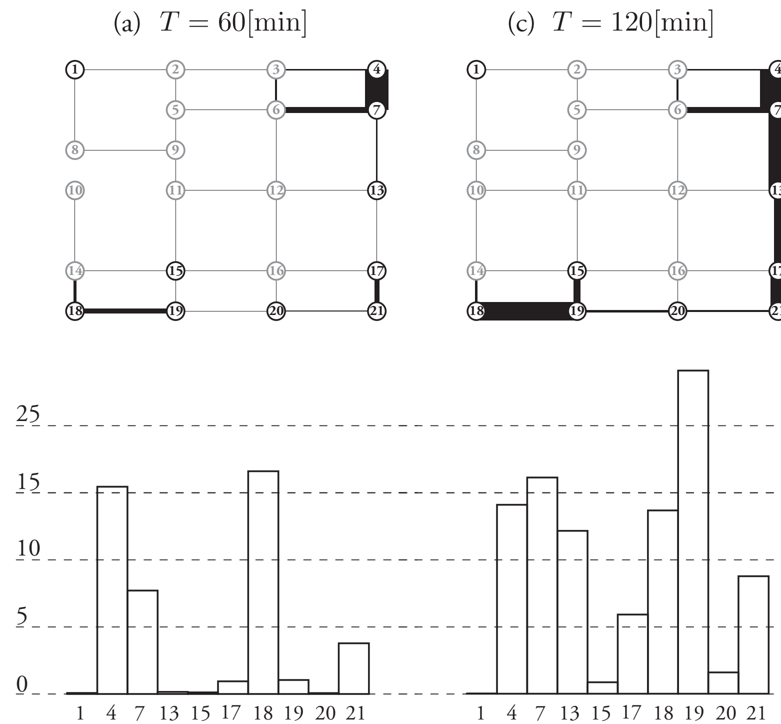


(a) (b) (c)

integrated modeling of route, activity place and duration choices.

Activity path modeling (ctd.)



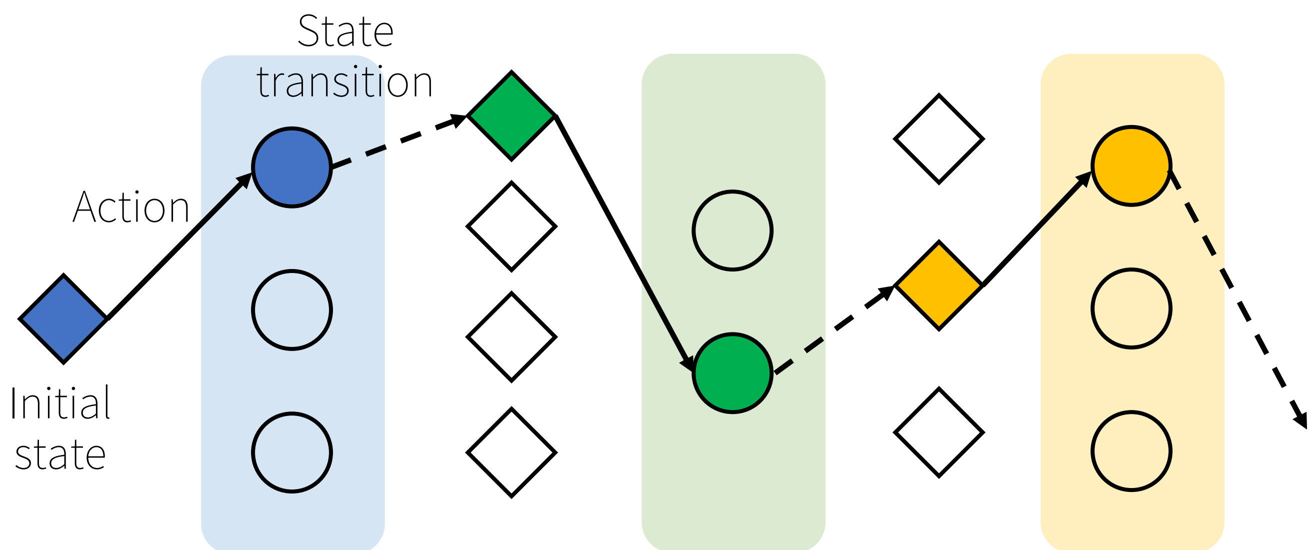


Upper: **pedestrian flows** on streets
Lower: ave. **activity duration** at places

(Appx.) Markov decision process (MDP)

To more generalize, define

- **Action**: choice behavior (what agent does)
- **State**: situation (where agent is in) that changes as result of action

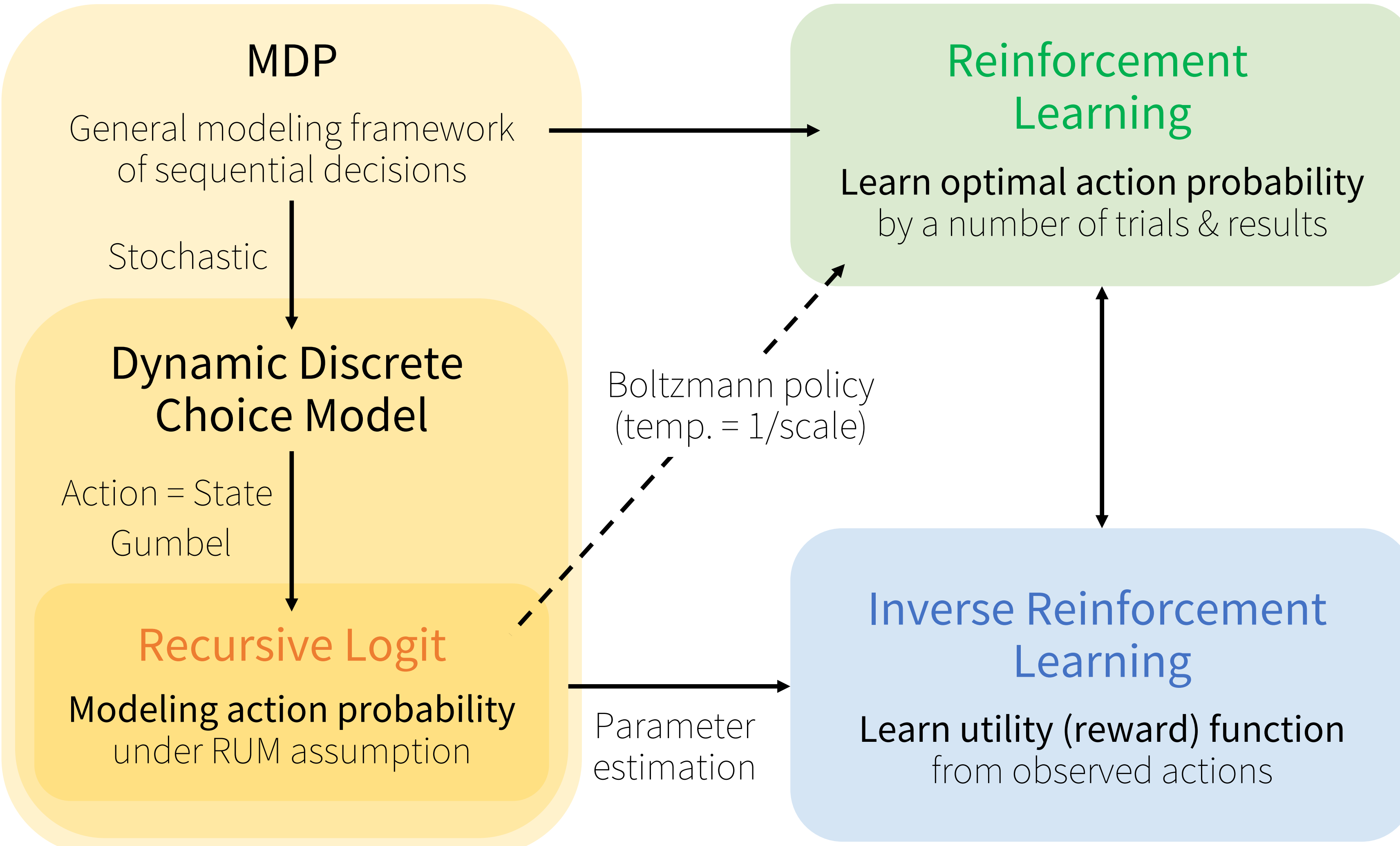


$$V(s) = \max_a \left\{ \sum_{s'} P(s'|s,a)\{v(s,a,s') + \gamma V(s')\} \right\}$$

State transition probability

*In route choice (recursive) modeling: **Action** is directly choice of **State**

(Appx.) Reinforcement Learning or Recursive Logit ?



To summarize,
Recursive Logit model

- is **computationally efficient**
  - No need to explicitly define choice set
- describes **dynamic sequential decisions**
  - Not only for route choice modeling
- shares **common mathematical foundations** with other state-of-the-art studies
  - Including machine learning

Major problem

RL model assigns high probabilities to **overlapping paths** due to

- IIA property of logit model
- considering all feasible paths including cyclic paths
  - assign flows (probabilities) on the same links many times
  - often cause computational intractability

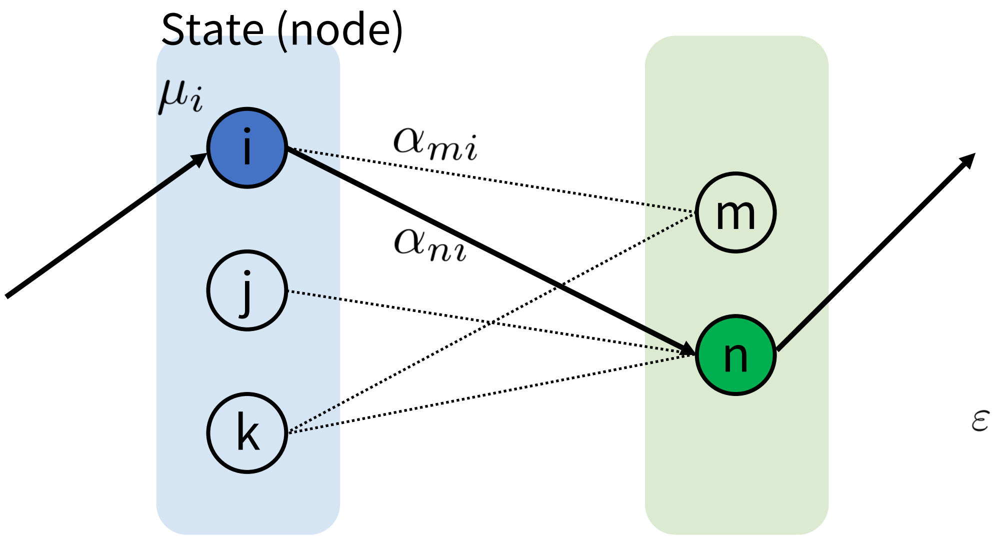
⇓

A solution is using:

- Network-GEV based model
- Time-space network with time constraint

Network-GEV route choice model

Directly translate a (state) network to a **GEV network**:



Multilevel cross-nested structure

$$\varepsilon(j|i) \overset{\text{i.i.d.}}{\sim} \text{Gumbel}(0, \mu_i), \forall j \in \mathcal{F}(i)$$

$$\sum_{i \in \mathcal{B}(j)} \alpha_{ji} = 1, \forall i \in \mathcal{S}$$

$$V^d(i) = \mathbb{E}\left[\max_{j \in \mathcal{A}(i)} \{v(j|i) + V^d(j) + \varepsilon(j|i) + \frac{1}{\mu_i^d} \ln \alpha_{ji}\}\right]$$

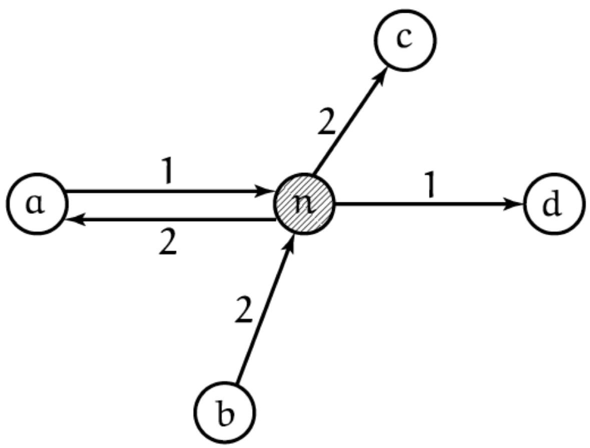
corrected to capture the correlation

$$\Leftrightarrow \quad e^{\mu_i^d V^d(i)} = \sum_{j \in \mathcal{A}(i)} \alpha_{ji} e^{\mu_i^d v(j|i)} \left(e^{\mu_j^d V^d(j)}\right)^{\frac{\mu_i^d}{\mu_j^d}}$$

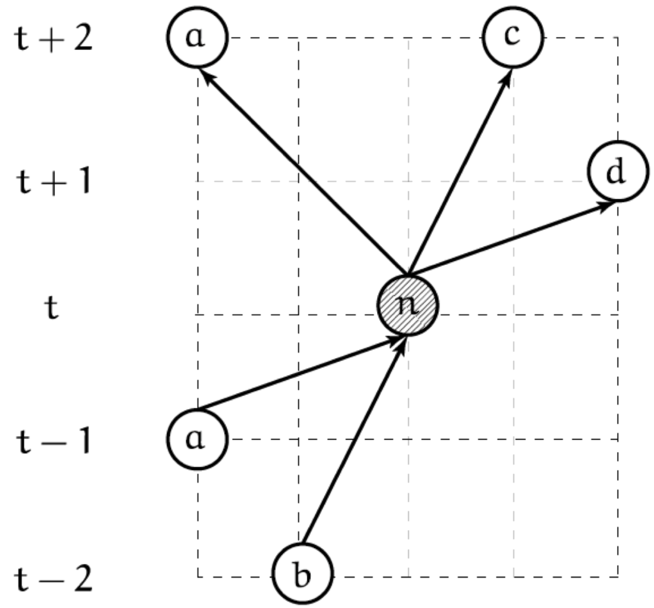
*system of **nonlinear** equations

Time-space network representation

is inherently **acyclic** and describes more **realistic** network structures.



(a) Spatial network

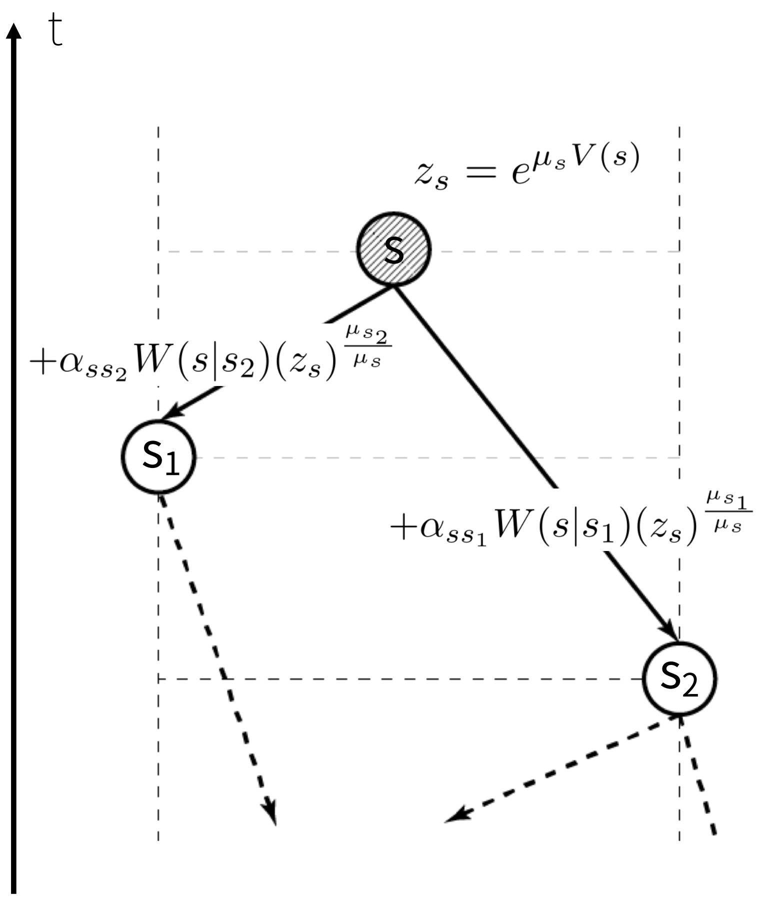


(b) Time-space network

	Spatial network	Time-space network
State:	**Node** (or link)	**Pair of time and node** (or link)
Connection:	$\mathcal{B}(n) = \{a, b\}$	$\mathcal{B}(t, n) = \{(t-1, a), (t-2, b)\}$
	$\mathcal{F}(n) = \{a, c, d\}$	$\mathcal{F}(t, n) = \{(t+2, a), (t+2, c), (t+1, d)\}$

Time-space network representation

is **simply computable** and **restricts network states** by terminal state.



Input: $\mathcal{S} = \{(T, d)\}, z(T, d) = 1$

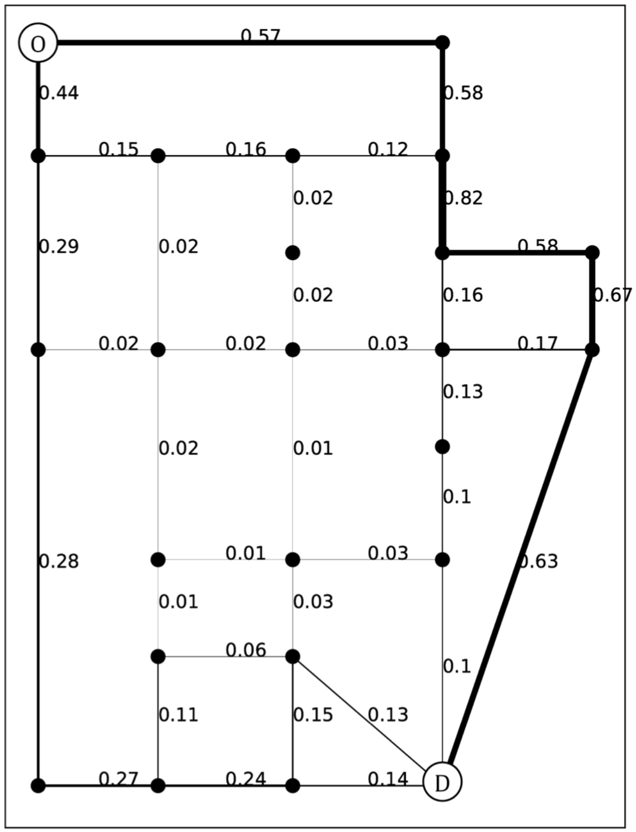
Output: $\mathcal{S}, \mathbf{z} \in \mathbb{R}^{|\mathcal{S}|}$ # terminal state (T, d)

$t = T$

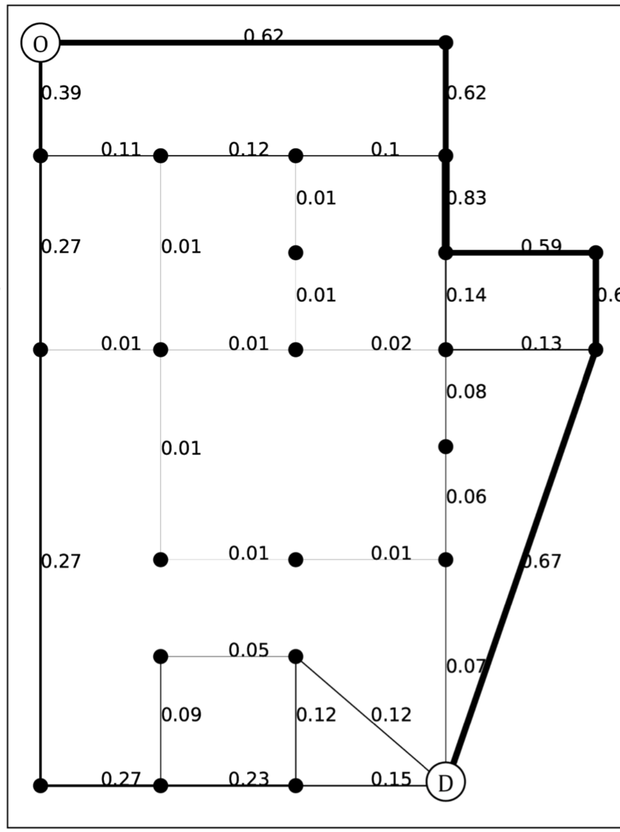
While True:

- for $s \in \mathcal{S}(t)$ # all states at t
  - for $s' \in \mathcal{B}(s)$ # all states connecting to s
    - if $s' \notin \mathcal{S}$ # if appearing for the first time
      - $\mathcal{S} := \mathcal{S} \cup s'$ # update the state set
      - $z(s') := 0$ # initialize the solution
    - $z(s') := z(s') + \alpha_{ss'} W(s|s')(z_s)^{\frac{\mu_{s'}}{\mu_s}}$ # recursively update the solution
  - end for
- end for
- $t := t - 1$
- if $t == 0$: break

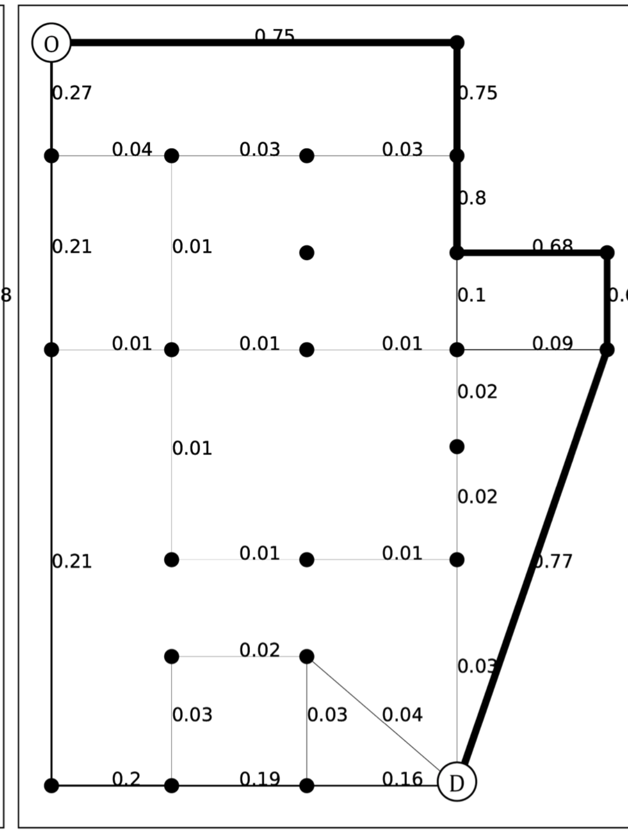
Logit



Logit with time constraint



Network-GEV with time constraint



- Counterclockwise way: more **overlaps** and logit overestimates the probability
- Without time constraint: more probabilities assigned to **cycles**

Ran out of time...

but we recently proposed an efficient **dual-type algorithm for Network-GEV based SUE**, which is also useful to:

- **Parameter estimation**
  - for entropy-constrained formulation
- **Dynamic pricing**
  - based on capacity-constrained assignment formulation

Markovian Traffic Equilibrium Assignment based on Network Generalized Extreme Value Model

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Abstract

This study establishes a novel framework of Markovian traffic equilibrium assignment based on the network generalized extreme value (NGEV) model, which we call *NGEV equilibrium assignment*. The use of the NGEV model in traffic assignment has recently been proposed and enables capturing the path correlation without explicit path enumeration. However, the NGEV equilibrium assignment has never been investigated in the literature, which has limited the practical

*available at arXiv!

Questions ?

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References

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- Fosgerau, M., Frejinger, E., Karlstrom, A., 2013. A link based network route choice model with unrestricted choice set. *Transportation Research Part B: Methodological* 56, 70–80.
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- Oyama, Y., 2017. A Markovian route choice analysis for trajectory-based urban planning. PhD thesis, The University of Tokyo.
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NGEV route choice model:

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> + optimization/solution algorithm:

- Oyama, Y., Hara, Y., Akamatsu, T., 2020. Markovian Traffic Equilibrium Assignment based on Network Generalized Extreme Value Model. arXiv:2009.02033.